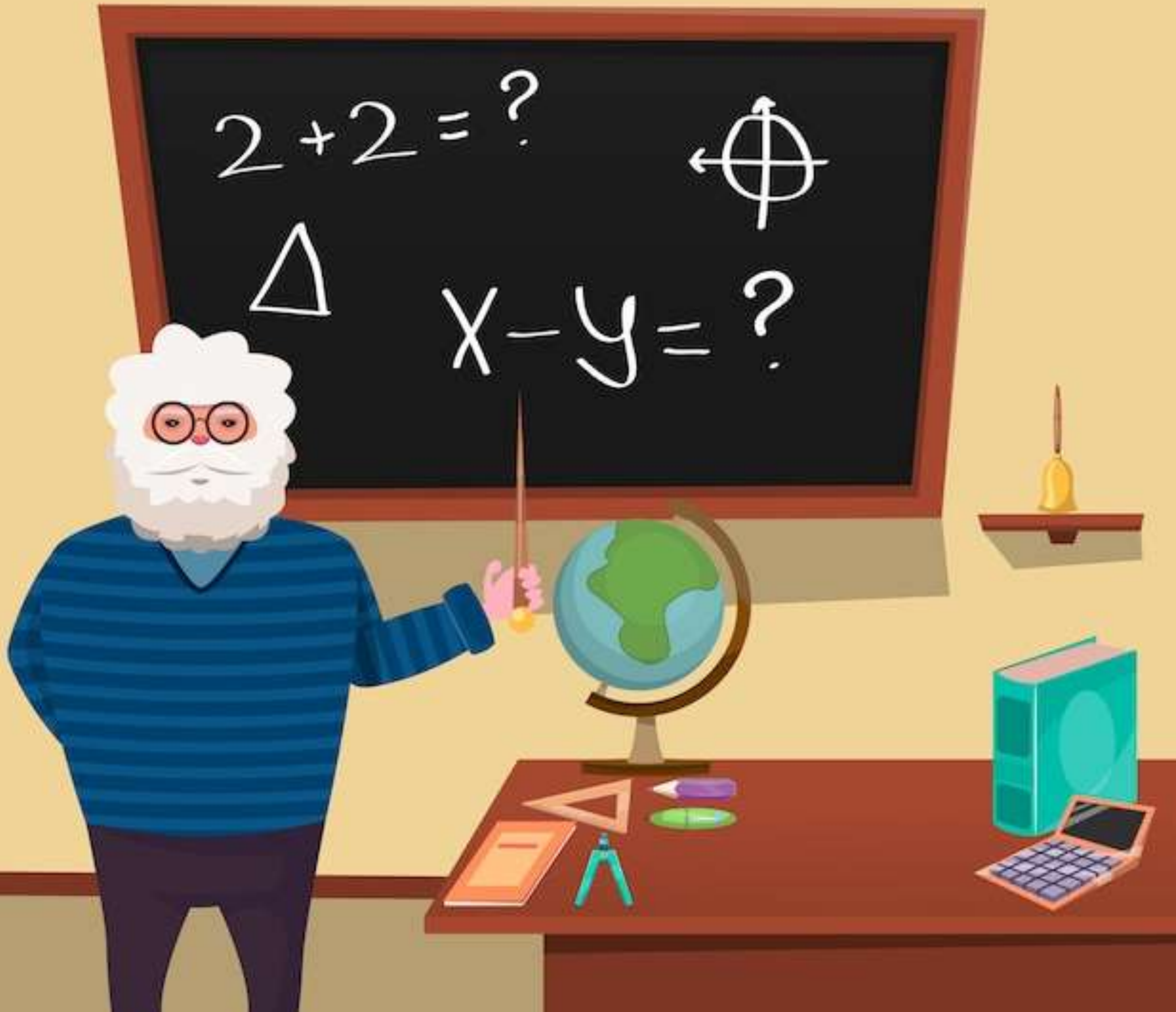
Proportion (Part 1)



► Definition

Two numbers are said directly proportional if a number increases, the other number also increases at the same rate.

Example:

I give private lessons and charge $10 per hour. How much does I get paid if I teach 5 hours?

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The rate of pay will be the same no matter how many hours she work.

↓

If the number of hours ↑ the amount she’s paid ↑

at the same rate.

▶ Definition

I give private lessons and charge $10 per hour. How much does I get paid if I teach 5 hours?

Then, $\frac{10}{1} = \frac{t}{5}$

$t = 5 \times 10 = 50\$$

In general, x and y are directly proportional if $\frac{\mathrm{y}}{\mathrm{x}} = \mathrm{k}$ constant i.e.

Algebraic relation

$\mathrm{y} = kx$

k is called constant of proportionality

► Table of proportionality

$\times 10$

x	1	2	3	4	5	10
y	10	20	30	40	50	100

$$y = 10x$$

x	5	8	12
y	30	48	72

$$\frac{30}{5} = 6 \ ; \ \frac{48}{8} = 6 \ ; \ \frac{72}{12} = 6$$

So table of proportionality

$$y = 6x$$

x	4	7	10
y	24	42	50

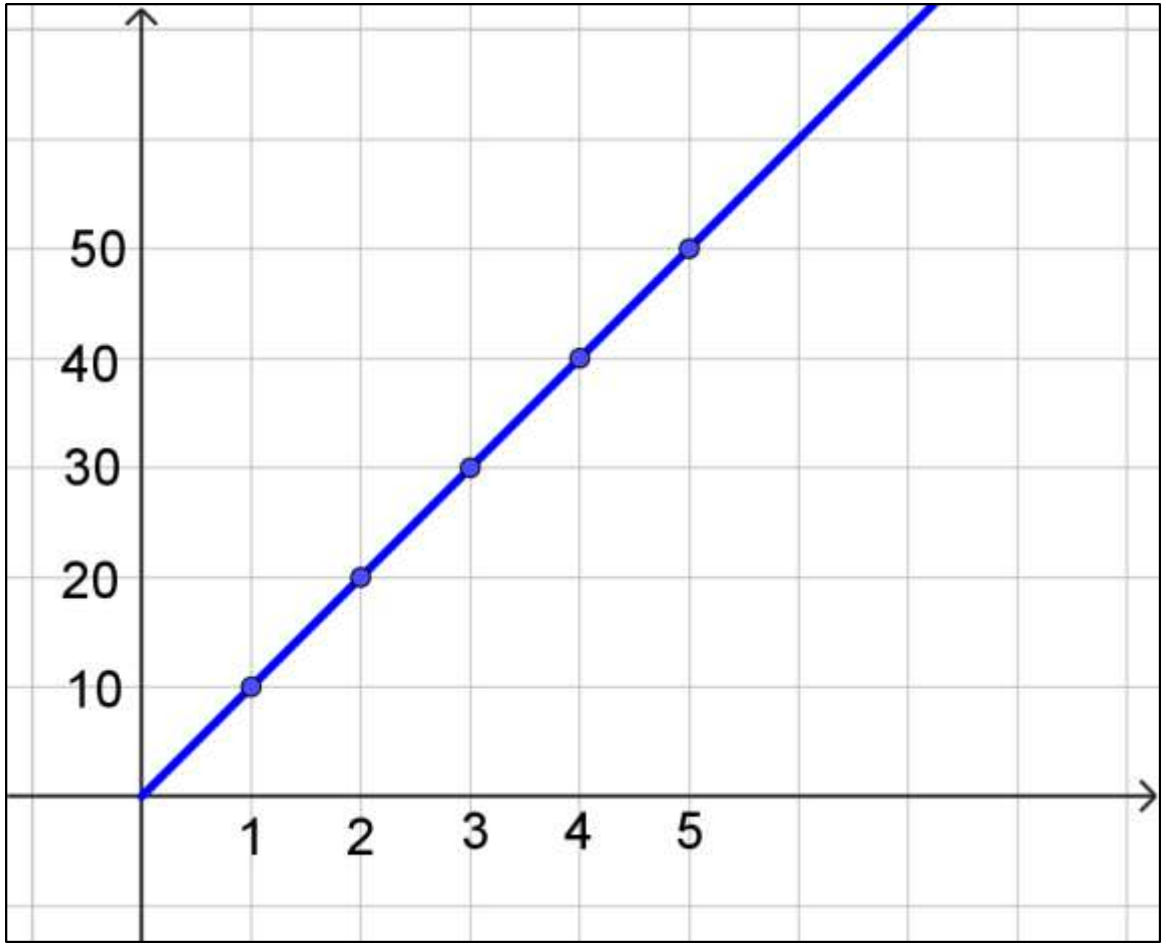
$$\frac{24}{4} = 6 \ ; \ \frac{42}{7} = 6 \ ; \ \frac{50}{10} = 5 \neq 6$$

So not table of proportionality

► Graphical representation

x	1	2	3	4	5
y	10	20	30	40	50

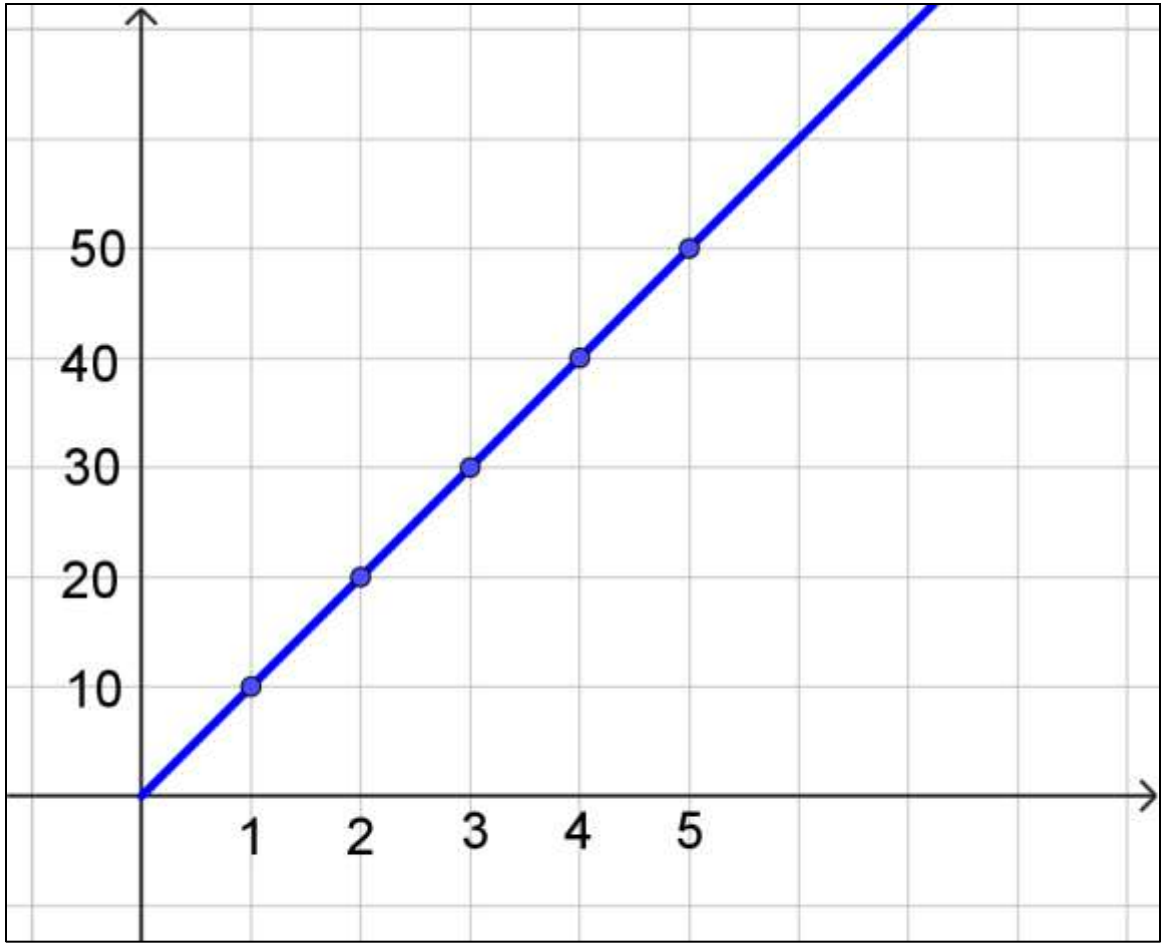
$y = 10x$



Straight line that passes through the origin

► Graphical representation

In general, we can represent a proportional relation of the form $y = ax$ in a system of axes by a line passing through the origin and the point $(1; a)$.
The relation $y = ax$ is called linear relation

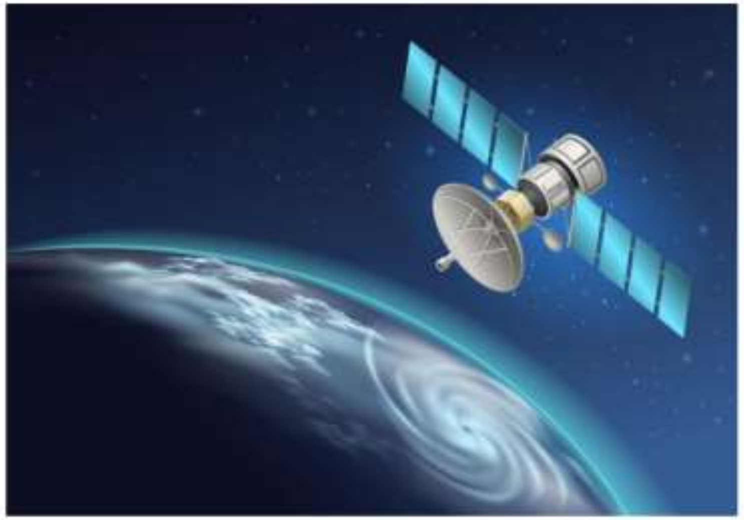


► Application # 1

The orbit of an artificial satellite is 10 000 km above the earth's surface. This satellite completes one turn in 2 days. Taking the radius of Earth to be 6400 km approximately:

a) What is the distance travelled by the satellite in one turn?

The distance is $D = 2\pi(6400 + 10000)$
$= 102992 \text{ km}$

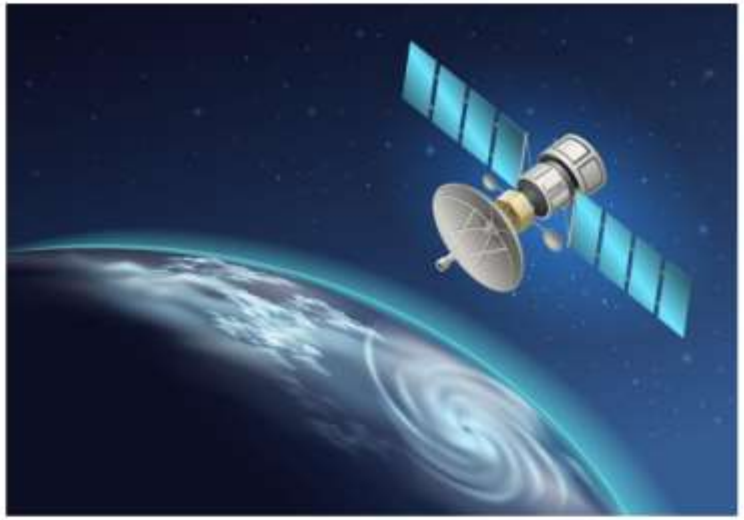


► Application # 1

The orbit of an artificial satellite is 10 000 km above the earth's surface. This satellite completes one turn in 2 days. Taking the radius of Earth to be 6400 km approximately:

b) What is the distance travelled by the satellite in one day?

102 992 ÷ 2 = 51 496 km

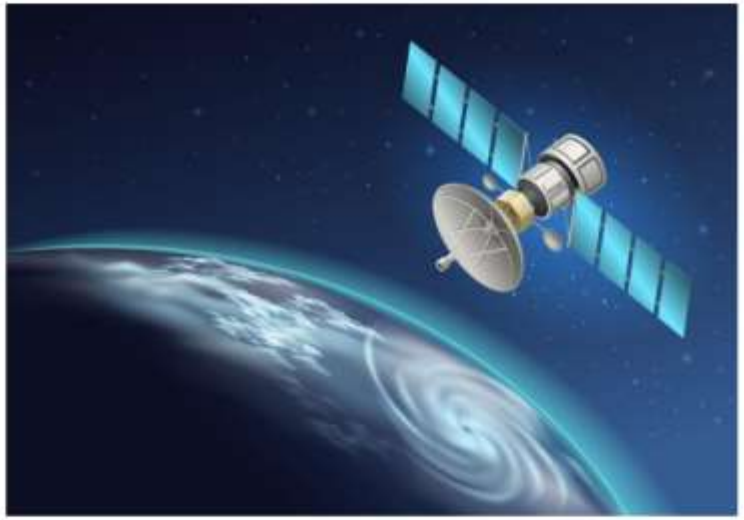


► Application # 1

The orbit of an artificial satellite is 10 000 km above the earth's surface. This satellite completes one turn in 2 days. Taking the radius of Earth to be 6400 km approximately:

c) How long does it take the satellite to complete 10 turns?

Turns	1	10
Days	2	x



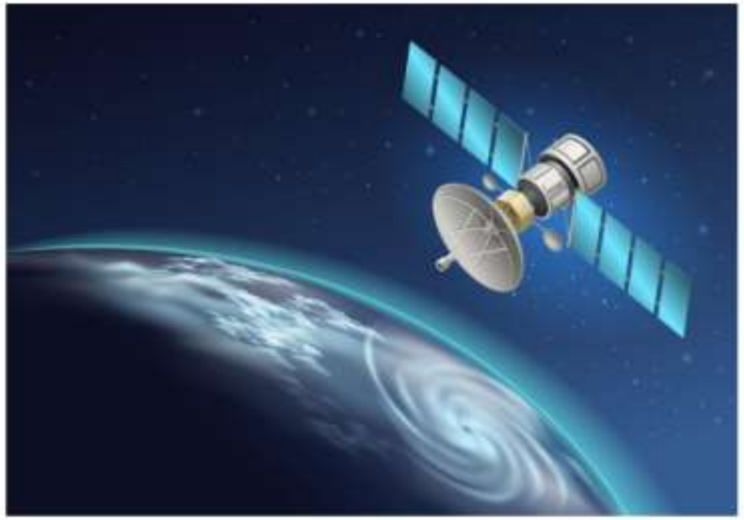
$$x = 2 \times 10 = 20 \text{ days}$$

▶ Application # 1

The orbit of an artificial satellite is 10 000 km above the earth's surface. This satellite completes one turn in 2 days. Taking the radius of Earth to be 6400 km approximately:

d) How many turns does it make in 18 days?

Turns	1	x
Days	2	18



$$x = \frac{1 \times 18}{2} = 9 \text{ turns}$$

► Application # 2

Three numbers a, b and c are directly proportional to 2,3 and 5. find a, b and c knowing that $a + b + c = 30$

$\frac{a}{2} = \frac{b}{3} = \frac{c}{5} = k$

$\frac{a}{2} = k \quad ; \quad a = 2k$

$\frac{b}{3} = k \quad ; \quad b = 3k$

$\frac{c}{5} = k \quad ; \quad c = 5k$

$a + b + c = 30$

$2k + 3k + 5k = 30$

$10k = 30$

$k = \frac{30}{10} = 3$

$a = 6 \quad ; \quad b = 9 \quad ; \quad c = 15$